2.4 Answers and solutions

2.4.1 Exercises

E71 $q = 5, \quad r = 1$
E72 $q = 33, \quad r = 3$
E73 $q = 6, \quad r = 0$
E74 $9 = 3 \cdot 3$. $21 = 3 \cdot 7$. $39 = 3 \cdot 13$. $51 = 3 \cdot 17$. $53 = 53$ (a prime number). $72 = 2^3 \cdot 3^2$. $88 = 2^3 \cdot 11$. $91 = 7 \cdot 13$.
E75 $18513 = 3^2 \cdot 11^2 \cdot 17$. $9288 = 2^3 \cdot 3^3 \cdot 43$. $103350 = 2 \cdot 3 \cdot 5^2 \cdot 13 \cdot 53$.
E76 False. **E77** False. **E78** True. **E79** True. **E80** False. **E81** True. **E82** False. **E83** True. **E84** False. **E85** False. **E86** True. **E87** False. **E88** True. **E89** True. **E90** False. **E91** True. **E92** True. **E93** True. **E94** True. **E95** True.
E96 1, 5, 25
E97 1, 2, 3, 6, 9, 18
E98 7, 14, 21, 28, ...
E99 23, 46, 69, 92, ...
E100 0 is the only multiple of 0.
E101 There are 25 such primes, namely 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
E102 Only 37, 101 and 173 are prime.
E103 10. **E104** 0. **E105** 14. **E106** 1. **E107** 44. **E108** 10. **E109** 1. **E110** 1287. **E111** 3. **E112** 1336. **E113** 30. **E114** 180. **E115** 6. **E116** 10176. **E117** 24. **E118** 35856. **E119** 58. **E120** 0. **E121** 0. **E122** 6. **E123** 1.
E124 No
E125 Yes
E126 The answer is $43^2 = 1849$
E127 7^3
E128 $2^4 \cdot 5 \cdot 41$
E129 $2 \cdot 11^2 \cdot 17$
E130 $3^3 \cdot 11 \cdot 37$
E131 $2^5 \cdot 3^4$
E132 $11^3 \cdot 13$
E133 11
E134 No
E135 Yes
E136 No
E137 $0, 9, 18, 27, \ -9, \ -18$
E138 $-21, 0, 21, 42, 63, 84$
E139 191
E140 18
E141 7 elements
E142 Yes

E143 No
E144 1
E145 6
E146 156
E147 12
E148 144
E149 Yes. **E150** Yes. **E151** Yes. **E152** No. **E153** Yes. **E154** Yes.

2.4.2 Problems

P7 To prove that this relation is reflexive, we must prove that a divides a for every a. That is, we must prove that for every a, there is an integer m such that $a = ma$. Obviously, the integer 1 has this property, for every a.
To prove that the relation is transitive, suppose that $a|b$ and $b|c$. Then, from the definition, there exist integers m_1 and m_2 such that $b = m_1 a$ and $c = m_2 b$. But this implies that

$$c = m_2 b = m_2 m_1 a$$

which shows that a divides c.
P8 Make a table, as we did before in class, with one column for the sum of the first n integers, and another column for the number $\frac{n(n+1)}{2}$. You will see that the two columns agree for $n = 1$, $n = 2$, $n = 3$, $n = 4$ and so on. In fact this formula is true for all n. The proof uses a method called induction, and we will prove the formula later.
P9 Again, you can compute the value of $n^2 + n + 17$ for small values of n. You will see that you get a prime number for $n = 0$, $n = 1$, $n = 2$, $n = 3$, $n = 4$, $n = 5$, $n = 6$, and also for small negative values of n. This might lead us to believe that we always get a prime number. However, try the number $n = 17$. Do we get a prime number in this case?