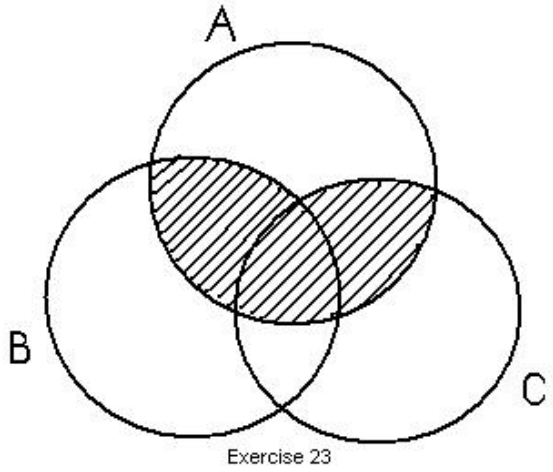
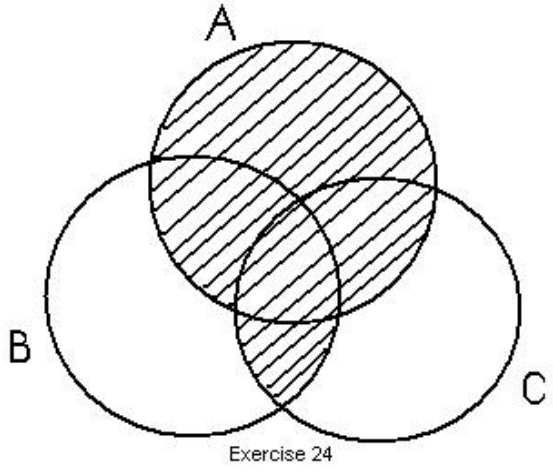
1.5 Anwers and solutions

1.5.1 Exercises E1 to E70

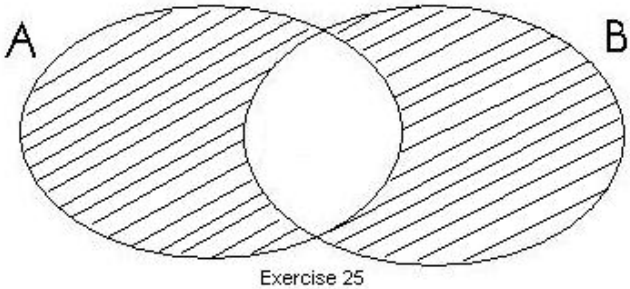
E1 False. **E2** True. **E3** False. **E4** True. **E5** False. **E6** False. **E7** True. **E8** $\{1, 3, 5\}$. **E9** $\{6\}$. **E10** $\{1, 2, 3, 4, 5, 6\}$. **E11** $\{(2, 6), (2, 7), (4, 6), (4, 7), (6, 6), (6, 7)\}$. **E12** $\{2, 4, 6\}$. **E13** $\{6\}$. **E14** $\emptyset$. **E15** $\{2, 4, 6\}$. **E16** $\emptyset$. **E17** 12 elements. **E18** 6 elements. **E19** Yes. **E20** Yes. **E21** No. **E22** Yes. **E23, E24, E25** In each exercise, draw the Venn diagrams of both sets, and check that they are equal. See the pictures. **E26** 2 elements. **E27** 5 elements.



Exercise 23



Exercise 24



Exercise 25

E28 12 elements. **E29** 4 elements. **E30** n^2 elements. **E31** 2 subsets. **E32** 4 subsets. **E33** True. **E34** False. **E35** True. **36** True. **E37** True. **E38** False. **E39** False (because $-1 \notin \mathbb{N}$). **E40** False. **E41** True. **E42** True. **E43** True. **E44** True. **E45** False. **46** True. **E47** True. **E48** True. **E49** No. **E50** Yes. **E51** No. **E52** Yes. **E53** No. **E54** 10 cells. **E55** 1 cell. **E56** Yes. **E57** No. **E58** No. **E59** $\mathbb{N}$. **E60** $\mathbb{N}$. **E61** $\{x \in \mathbb{N} \mid x \geq 2\}$. **E62** No (because $f(0) = f(1)$). **E63** No (because 2 is not in the image). **E64** No. **E65** $\mathbb{N}$. **E66** 1, 3, 7, 13 (for example). **E67** 73. **E68** 45. **E69** No (because $f(0) = f(1)$. **E70** Yes.

1.5.2 Problems P1 to P6

The problems were discussed in the tutorial class. Here we just give brief answers and some hints.
P1 Answer: 243 different functions. In general, if A has m elements and B has n elements, then there are n^m different functions from A to B.
P2 One example is the following relation on the set of integers:

$$a \; R \; b \quad \text{if} \quad |a - b| \leq 1$$

Verify that this relation is reflexive and symmetric, but not transitive.
P3 Try to first answer the question for $n = 1$, $n = 2$, $n = 3$. Try to guess a pattern. Then try to prove your guess.
P4 Yes.
P5 Yes.
P6 Answers: (i) 21. (ii) No. For example, $f(10) = f(1)$. (iii) Yes. If $n \in \mathbb{N}$ is any given number, we can consider the number

$$a = \underbrace{111\ldots1}_{n \text{ times}}$$

Clearly, we have $f(a) = n$.
(iv) No. (v) 2. (vi) 5. (vii) $\mathbb{N}$. (viii) $\mathbb{N}$. (ix) No. (x) Yes, see problem 5 above.